

Preface

This is a book on coupling, including self-contained treatments of stationarity and regeneration. Coupling is the central topic in the first half of the book, and then enters as a tool in the latter half. The ten chapters are grouped into four parts as follows:

Chapters 1–2 form an **introductory** part presenting basic elementary couplings (Chapter 1 on random variables) and the classical triumphs of the coupling method (Chapter 2 on Markov chains, random walks, and renewal theory).

Chapters 3–7 present a general **coupling** theory highlighting maximal couplings and convergence characterizations for random elements, stochastic processes, random fields, and random elements under the action of a transformation semigroup.

Chapters 8–9 present Palm theory of **stationary** stochastic processes associated with a simple point process. Chapter 8 treats the one-dimensional case and Chapter 9 the higher-dimensional case.

Chapter 10 deals with **regeneration**, both classical regenerative processes and three generalizations: wide-sense regeneration (as in Harris chains); time-inhomogeneous regeneration (as in time-inhomogeneous recurrent Markov chains); and taboo regeneration (as in transient Markov chains). It ends with a section on perfect simulation (coupling from-the-past). This enormous chapter is thrice the size of a normal chapter, and is really a book within the book.

For more information on the content of the book, see the introductions to the chapters. Also, the table of contents provides a structural review.

The book should be of interest to students and researchers in probability, stochastic modelling, and mathematical statistics. It is written with a Ph.D. student in mind, and the first two chapters can be read at the master's level and even at an advanced undergraduate level.

The book is mathematically self-contained, relying only on basic measure-theoretic probability. Measure-theoretic language is suppressed in the first two chapters, and then enters heavily in Chapter 3 to be used explicitly for the rest of the book; Ash (1972) is used as reference, but Billingsley (1986) is also fine. Some prior knowledge of elementary Markov chain theory would be useful, at least in Chapter 2; Karlin and Taylor (1975) and Cinlar (1975) are excellent, and the compact first two sections of the first two chapters in Asmussen (1987).

Some Conventions

In order to make clear what results belong to the measure-theoretic background, the term 'Fact' is used for results stated without proof, while the terms 'Theorem' and 'Lemma' are reserved for results that are proved here. Facts of basic importance throughout the book are restricted to Chapter 3 (Sections 3 and 4).

Sections are enumerated within chapters. For instance, the 4th section in the 3rd chapter is referred to within the chapter only as 'Section 4'; but in the other chapters as 'Section 4 in Chapter 3' or 'Chapter 3 (Section 4)'.

Subsections are enumerated within chapters and sections: the 5th subsection of the 4th section in the 3rd chapter is referred to within the chapter as 'Section 4.5'; but in the other chapters as 'Section 4.5 in Chapter 3'. The same goes for Theorems, Lemmas, Facts, Remarks, and Figures.

Definitions are stated in the text, and only indicated by writing the *concept* being defined in italics (we also use italics for emphasis). Figures are placed in the text precisely where they should be consulted (mostly), but the text does not rely on them. We use both parenthesis () and brackets [] for comments that can be skipped. Historical and bibliographic notes are deferred to a separate section at the end of the book.

The symbol X (and X' , \tilde{X} , X_1, \dots) is reserved for real-valued random variables. The symbol S_k always denotes a sum, $S_k = S_0 + X_1 + \dots + X_k$. On the other hand, S is either a sequence of S_k (one-sided sequence in Chapters 2, 3, and 10; two-sided in Chapter 8), or a d -dimensional random vector (Chapters 7 and 9). The symbol U is reserved for a variable uniform on $[0, 1]$. The symbol Z is reserved for processes. The symbol Y often denotes a random element in a general space; and $\mathbf{P}(Y \in \cdot)$ is the distribution of Y .

Errors are bound to abound in the book, in spite of all the thinning attempts. For errata, and even some notes and references, see my homepage (www.hi.is/~hermann) or Springer's (www.springer-ny.com). If you find an error or have a comment, please send me an informal note by e-mail (To: hermann@hi.is; Subject: book).

Acknowledgements

It took four long years to write this book, word for word, from the first word on the first page to the last word on page four-hundred-seventy-eight. Previously, the book had been in preparation for five years; and before that, subconsciously for years.

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And now, as this book goes to print, I am informed that I have been awarded the Ólafur Daníelsson Prize in Mathematics for my research, most of which can be found in some form in this book. I am deeply moved and will use the generous sum to recover from this work, and prepare for the next.

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Hermann Þórisson