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Notation

$\mathbb{R} = (-\infty, \infty)$ and $\mathbb{R}_+ = [0, \infty)$
 $\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$ and $\mathbb{Z}_+ = \{0, 1, \dots\}$
 $X \stackrel{D}{=} Y$ = X and Y have the same distribution
 $a := b = a$ is defined to be b
 $f \equiv g$ = for all x it holds that $f(x) = g(x)$
 1_A = the indicator function of the set A
 $\#A$ = the number of elements in the set A
 $a \vee b, a \wedge b$ = maximum and minimum of a and b
 a^+, a^- = $a \vee 0$ and $-a \wedge 0$
 $[x] = \inf\{n \in \mathbb{Z} : n \leq x\}$ = integer part of x
 $x \bmod h = x - [x/h]h$ = the remainder when x is divided by h
 $f(x-) = \lim_{y \uparrow x} f(y)$ = left-hand limit
 $\mu \wedge \lambda$ = the greatest common component of the measures μ and λ
 $(\mu - \lambda)^+, (\mu - \lambda)^-$ = positive and negative parts of $\mu - \lambda$
 $\mu \perp \lambda$ = the measures μ and λ are mutually singular
 $\mu|_{\mathcal{A}}$ = restriction of the measure μ to the sub- σ -algebra \mathcal{A}
 \xrightarrow{D} = convergence in distribution
 \xrightarrow{tv} = convergence in total variation
 $\|\cdot\|$ = the total variation norm
dot-convention: $f(\cdot) = f$ = the function with value $f(x)$ at x , for all x
 $\sigma(\cdot)$ = the σ -algebra generated by \cdot
i.i.d. = independent and identically distributed